



Thinning-response modifier term in growth models: An application on clonal *Tectona grandis* Linn F. stands at the amazonian region

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ABSTRACT

Thinning is a widely used silvicultural method in the *Tectona grandis* Linn. F. (teak) stands, however, limited work has been done on modeling for predicting both short- and long-term responses to this treatment. Dynamic growth models for teak stands in Easter Amazon were developed and evaluated from data of 67 plots measured between two and ten years old. We tested the models of Chapman-Richards, Hossfeld, and Lundqvist-Korf using the generalized algebraic difference approach (GADA) to describe the basal area of the stand, including the effect of thinning, which was quantified from a silvicultural treatment response function. Dominant height was described using a dynamic equation derived onwards by Lundqvist-Korf model, considering one parameter to be site specific, while the stand volume was described as of Chapman-Richards model using the theory of covariates. We used 80% of the data for fitting and 20% for validating the developed models. The following fitting statistics were used: Akaike's information criterion, square root of the mean percent error and estimation efficiency. For the validation, was used the equivalence test (regression based TOST using bootstrap). The equation derived from the Chapman-Richards function from GADA integrated with a thinning modifier provided the best performance in describing stand dynamics. The developed thinning modifier were shown to significantly improve predictions of stand-level basal area growth, and the results showed that models that disregard silvicultural treatment cannot be used to obtain reliable projections of the stands under analysis. The growth models provided detailed and accurate description of the dynamics of teak stands in the Amazon region, allowing for a well-defined maximum volume rotation. Overall, the study highlights the importance of including thinning modifiers in growth and yield models, especially for shade intolerant species, such as teak.

1. Introduction

Tectona grandis Linn. F. (teak) is one of the world's most valuable tropical woods and is desired globally for its beauty, stability, natural strength, and extensive use (Tewari et al., 2014; Tewari and Singh, 2018; Yang et al., 2020; Yasodha et al., 2018). Teak is found in over 60 tropical and subtropical countries, and there are an estimated 6.8 million hectares planted worldwide, of which 83% are found in Asia (Barrantes-Madrigal et al., 2021; Kollert and Kleine, 2017; Yasodha et al., 2018). Between 2005 and 2014, international trade in teak roundwood totaled US\$487 million, accounting for approximately 3% of the total capital generated by the international timber market (Midgley

et al., 2015). Given the recent export ban imposed by Myanmar, there has been a sharp increase in teak price, especially from commercial plantations, which has led to the expansion of the planted area in Latin America and Africa, attracting large private sector investments (Kollert and Kleine, 2017; Yasodha et al., 2018). This has induced progress in cultivation of the species, such as by the development of new clones, suitable methods for soil management, and gains in productivity (Tondjo et al., 2018). However, it is necessary to adopt modern silvicultural and management techniques to increase the production and productivity of plantations to ensure the supply of high-quality wood, since only 2.0/2.5 million m³ of the total 30 million m³ produced annually are harvested in natural and planted forests (Midgley et al.,

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2015). This level of production is expected to increase, particularly in planted forests in Central and South America.

Teak plantations in Brazil have been concentrated mainly in the central-western and northern regions of the country, predominantly in the Amazon, which corresponds to 94,000 ha (IBA, 2019). Despite the expansion of plantations, Brazilian cultivation is still in the incipient stage when compared to that of the *Pinus* and *Eucalyptus* genera (Vendruscolo et al., 2019). Therefore, effective management of teak, or any forest typology, is only possible when reliable information about present and future forest conditions is available. This is especially important in intensively managed forest stands where thinning is required (García et al., 2011; Tewari and Singh, 2018), such as in teak stands. Some studies have shown that thinning produces significant changes in the growth of teak plantations (Seta et al., 2021), and can impact important ecosystem services, such as water production (Sinacore et al., 2019). Therefore, to assist decision-making and support silvicultural treatments, teak growth and production should be characterized, to consolidate its cultivation in Brazil, and thereby raise its production to competitive levels in the international market (Vendruscolo et al., 2019).

Consequently, the proposition of growth models must consider thinning as an important practical silvicultural strategy, as its prediction and prognosis can ensure a choice of management regimes that enable economic success and important ecosystem service maintenance (Gyawali and Burkhart, 2015; Sinacore et al., 2019). However, methods for quantifying the effect of silvicultural treatments in growth and yield models are not straightforward and are often limited by the type of information available (Weiskittel et al., 2011). In addition, no recent studies on teak have considered thinning as a state-modifying variable (Tewari et al., 2014; Tewari and Singh, 2018; Torres et al., 2020).

Many studies have been conducted to incorporate the effect of thinning on growth and yield models. The main challenge is quantifying the effect of the practice on the stand as a whole, which is characterized by the use of thinning modifier terms or thinning response functions (Gyawali and Burkhart, 2015; Kuehne et al., 2016; Weiskittel et al., 2011). Several approaches to include the effect of thinning in forest models have been described by Söderbergh and Ledermann (2003), ranging from empirical to process-based. These modifiers can be additive or multiplicative (with the latter being preferred for several reasons, such as greater flexibility and accuracy), and depend on a number of factors, such as the age at which the treatment occurred and the thinning method used (Kuehne et al., 2016; Weiskittel et al., 2011). Although several studies have developed thinning modifiers, most have focused on conifer or temperate species, including those presented by Pienaar and Shiver (1986) for *Pinus elliottii* Engelm.; Gyawali and Burkhart (2015) for *Pinus taeda* L.; and Kuehne et al. (2016) for the temperate climate species *Abies balsamea* (L.) Mill, *Pinus strobus* L., and *Thuja occidentalis* L.

The effect of thinning on remaining forest growth is usually associated with basal area projection models, as they are key variables at the total stand level, since basal area is directly related to other variables of economic importance, such as total volume (Barrio Anta et al., 2006; Castedo-Dorado et al., 2007; Prada et al., 2019; Torres et al., 2020). Estimates from basal area growth equations can be used to quantify competition indices, which are used in individual tree models and form a link between explicit and implicit models (Castedo-Dorado et al., 2007; Gadow et al., 2007; Hein and Dhôte, 2006; Martins et al., 2014; Prada et al., 2019). Consequently, growth functions for stand basal area must meet important assumptions to represent biological growth, such as biological logic, polymorphism, inflection and asymptote point, compatibility, and parsimony (Castedo-Dorado et al., 2007; Prada et al., 2019; Salekin et al., 2020). Such functions should present an asymptotic point when the projection age approaches infinity and they should be compatible, whereby the projections should be equal for the same growth period, i.e., the result of the projection from t_0 to t_1 , and from t_1 to t_2 , should be the same as that from t_0 to t_2 . Finally, models should be parsimonious, as models that are too complex and include too many

variables can lead to instabilities and inaccuracies (Prada et al., 2019; Salekin et al., 2020).

As managed forests are known to be dynamic biological systems that change in response to the environment and silvicultural practices (Salekin et al., 2020), static forest growth models such as yield tables or cumulative growth functions are unsuitable for detailing these systems. The generalized algebraic difference approach (GADA) can be applied to model the growth of any site-dependent variable, using self-reference with theoretical variables (Cieszewski and Bailey, 2000; Cieszewski and Strub, 2008). This approach has been employed to model dominant height, basal area, stand volume, trees per unit area, biomass, and carbon sequestration (Prada et al., 2019). However, its use in Brazil is limited, and no application for modeling teak growth and yield has yet been performed, while in Latin America this method has focused on modeling the dominant height of the species (Cañadas et al., 2018).

Some studies have explored the use of the GADA in basal area modeling, and indicated that it was not necessary to directly quantify thinning in the equations (Barrio Anta et al. 2006; Castedo-Dorado et al. 2007; Weiskittel et al. 2011). However, that research was conducted on *Pinus radiata* D. Don and *Pinus pinaster* Ait. which do not demonstrate differences in growth patterns between thinned and unthinned stands (Barrio Anta et al. 2006; Castedo-Dorado et al., 2007). No studies have explored the effectiveness of thinning modeling for teak, especially when using thinning modifier terms associated with the GADA. In addition, information on modeling teak growth and production in thinned plantations is still scarce (Torres et al., 2020).

The development of an approach that considers the precepts of forest biological growth and the effect of silvicultural treatments originating from thinning, is indispensable for predicting changes in the growth and production of forest stands. Therefore, we raise the following questions: Does the GADA guarantee accurate and trend-free adjustments without considering the effects of thinning on growth? Does the insertion of the modifier term in GADA models enable meaningful, accurate, and biologically consistent fits? For the first question, we formulated the hypothesis that, although GADA models are efficient in modeling growth and production, it is necessary to quantify their effects on the growth equations for stands that undergo thinning, especially for species that are sensitive to thinning. As for the second question, by including a thinning modifier term in the GADA models, significant coefficients are presented that allow for accurate and biologically consistent estimates for thinned and unthinned stands. To answer these questions, the main objective was to develop a biologically consistent models at the total stand level for teak forest plantations undergoing thinning. The specific objectives were to: (1) develop a response modifier for thinning at the total stand level, (2) integrate the response function into a GADA model, and (3) evaluate the significance of the thinning modifier on growth and yield estimates.

2. Methodology

2.1. Study area

Clonal stands of *Tectona grandis* Linn. F, were used in permanent circular plots with an area of 500 m² at an initial effective area per tree of 16.00 m², 14.06 m² and 12.25 m², totaling, on average, an initial density of 625, 711 and 816 trees per hectare, approximately. The stands were 10, 8, 7, 6, and 5 years of age (measured from 2 years) and located in Capitão Poço, PA, on property belonging to the company Tiête Agrícola Ltda. The farm has an area of approximately 2,400 ha, between the geographic coordinates 2°30'00" S, 47°20'00" W and 2°20'00" S; 47°30'00" W, of which 754.4 ha are represented by forest plantations. In this region, the domain of the dense ombrophylous forest predominates (IBGE, 2012), with mainly latosol and plinthosols (Embrapa, 2018). The relief is characterized as flat to gently undulating (IBGE, 2012). The climate of the region is type Am, according to the Köppen classification system, indicating a hot and humid tropical rainy climate with a short

dry season (Alvares et al., 2013).

The silvicultural practices in all stands consisted of controlling leaf-cutting ants with ant baits, clearing plantations with a bulldozer, liming with dolomitic limestone (3 t ha⁻¹), fertilizing with 200 g plant⁻¹ of NPK 8-28-16, and 100 g plant⁻¹ of KCl, controlling weeds by hoeing or mechanized and semi-mechanized weeding with a hydraulic tractor, maintaining by fertilization with application of boron (7 g plant⁻¹) and KCl (100 g plant⁻¹) and pruning with a saw and pole pruner. The first and second thinning (systematic or selective under thinning) were performed at 4.5 and 8.5 years (mean value), respectively, with an intensity of 50% reduction in basal area.

The selective thinning from below was constituted by the removal of suppressed, defective, and termite-infested individuals to favor the development of trees with larger dimensions and better stem qualities. Of the total number of plots sampled, 15% were subjected to selective thinning. Systematic thinning removed individuals proportionally in all diameter classes following a regular spatial arrangement. The determination of selective thinned individuals occurred in alternate, parallel lines for the first thinning, which was systematic thinning. For both thinning events, the average intensity was approximately 50%. For the second thinning, 100% of the sample plots were subjected to selective thinning only.

The volume per tree sampled in each plot was obtained from the Takata model, whose equation was adjusted from the database provided by the company. Thus, the total volume was obtained by summing the volume of individual trees, which was extrapolated to hectare in each of the sampled plots:

$$V_i = DBH_i^2 H_i / (20510.8 + 286.7 DBH_i) \quad (1)$$

where V_i is the individual volume (m³); DBH_i is the Diameter at Breast Height (cm) and; H_i is the total height of the tree (m). $RMSE_{\%} = 5.77$ and $R_{yy}^{\sim} = 0.994$.

The data used for modeling were from successive measurements of 67 permanent plots, and included: the age of the stand (t), number of trees per hectare (N), volume estimates per hectare (V), basal area (G), dominant height by Assman methodology (Assmann, 1970) (h_{dom}), percentage of thinned trees ($N_t\%$), and the age at which thinning occurred (t_t). The database was divided into two random groups covering all ages and yield classes (Table 1): a database to fit the models (80% of total plots, 54 plots), and data to validate the models (20% of total plots–13 plots) (Gujarati and Porter, 2008).

2.2. Basal area modeling

Theoretical growth functions were selected as candidates for basal area growth modeling (Burkhart and Tomé, 2012) (Table 2). The selected functions possessed all desirable attributes for representing biological growth, such as inflection point, asymptote, and biological realism (Burkhart and Tomé, 2012; Salekin et al., 2020). Based on these equations, several dynamic models were formulated using the GADA to develop the projection function.

The GADA method consists of replacing certain parameters of the base model with explicit functions of x , which is an unobservable

independent variable that describes site productivity, such as soil conditions or ecological and climatic factors. Therefore, we followed the following procedures: (1) selection of a function to model the variable of interest; (2) definition of the parameters that will vary as a function of x , and expression of this relationship using a mathematical equation; (3) solution of the equation for x ; and (4) input of the solution to obtain the dynamic model for the variable of interest ($Y = f(t, x)$), given the initial conditions of t_0 and Y_0 . The two-dimensional basic equation ($Y = f(t)$), thereby expands into a three-dimensional equation ($Y = f(t, x)$) that describes both the cross section and longitudinal changes with the two independent variables, t and x . Since x cannot be reliably measured or even functionally defined, the final step in the GADA involves replacing x with the equivalent initial conditions t_0 and Y_0 ($Y = f(t, t_0, Y_0)$) (Cieszewski and Bailey 2000).

As a general notational convention, A , B and Γ were used to denote parameters in base models, while α , β and γ were used for global parameters in subsequent GADA formulations (Tewari et al., 2014).

The models presented in Table 2 are multiple polymorphic asymptotes. Model M1 expresses the asymptote as an exponential function of the theoretical variable x and the shape parameter as a linear function of the inverse of x . Model M2 was derived by Cieszewski and Strub (2008) from the Hossfeld function, with the coefficient A as a function of the summation of a constant with the theoretical variable x , and B as β/x . The M3 model was derived based on the Lundqvist-Korf function, considering both parameters A and B to be x -dependent, by arranging the base model in a form more suitable for manipulation of these two parameters, using the exponential of x instead of A . The parameter B was expressed as a linear function of the inverse of x .

2.3. Thinning effect on basal area growth

The effect of thinning on growth was modeled using a modified equation that responds to silvicultural treatment. The function was added as a multiplier in the base model presented in Table 2 ($\hat{y} = f_{base} \times f_{thin}$) (Gyawali and Burkhart, 2015; Kuehne et al., 2016). We followed this approach for both thinned and unthinned plots. The thinning response function for the basal stand area presented here is a modification of the function developed by Pienaar and Shiver (1986).

$$f_{thin} = \begin{cases} 1 & N_t = 0 \\ \delta \left[h_{dom_t} \left(\frac{N_t t_t}{N_a t_a} \right) \right] & N_t > 0 \end{cases} \quad (2)$$

Many studies have been conducted to incorporate the effect of thinning into growth and yield models, however, some of these functions include many interactions between independent variables, besides presenting many parameters (Weiskittel et al. 2011; Kuehne et al., 2016). Thus, we sought to develop a parsimonious thinning response function, because models that are too complex may be unstable and have poor predictive ability, and may not converge per iteration. To control the thinning intensity from the basal area, whether selective or systematic, we used the function presented by Field et al. (1978):

Table 1
Descriptive statistics for the variables of clonal teak stands, planted in Capitão Poço, Pará, Eastern Amazon.

Variables	Fit data				Validation data			
	Min.	Max.	Mean	SD	Min.	Max.	Mean	SD
T	24	120	50.16	20.12	24	120	57.32	21.05
N	140	820	551.32	153.45	180	820	546.15	209.84
h_{dom}	7.24	19.91	14.32	1.72	7.28	19.22	14.13	2.65
G	3.83	24.98	13.28	2.99	4.05	23.82	11.67	3.90
V	13.61	205.11	92.77	24.32	14.16	194.08	91.68	33.39
N_t (%)	2.27	70.27	48.64	17.90	41.17	64.71	50.47	5.46
t_t	60	108	68.42	12.41	60	108	69.60	16.48

Table 2

Base models selected for modeling basal area growth in clonal teak stands planted in Capitão Poço, Pará, Eastern Amazonia.

Model	Base model	Parameters related to the target variable x	Initial Solution for x with Y_0 and t_0	Dynamic equation
M1	Chapman-Richards: $Y = A[1 - \exp(-Bt)]^\Gamma$	$A = \exp(x) \quad \Gamma = \beta + \frac{\gamma}{x}$	$x_0 = \frac{1}{2}(\ln(Y_0) - \beta l_0 + r_0)$ With : $r_0 = \sqrt{(\ln(Y_0) - \beta l_0)^2 - 4\gamma l_0} \quad l_0 = \ln(1 - \exp(-\alpha t_0))$	$Y = Y_0 \left[\frac{1 - \exp(-\alpha t)}{1 - \exp(-\alpha t_0)} \right]^{(\beta + \gamma/x_0)}$
M2	Hossfeld: $Y = \frac{A}{1 + Bt^{-\Gamma}}$	$A = \alpha + x \quad B = \frac{\beta}{x}$	$l_0 = (Y_0 - \alpha)^2 + r_0 \quad r_0 = 4\beta Y_0 t_0^{-\gamma}$	$Y = \frac{\alpha + x_0}{1 + \beta/x_0 t^{-\gamma}}$
M3	Lundqvist-Korf: $Y = A \exp(-Bt^{-\Gamma})$	$A = \exp(x) \quad B = \alpha + \frac{\beta}{x}$	$x_0 = \frac{1}{2}[\ln(Y_0) + r_0]$ With : $r_0 = \sqrt{\ln(Y_0)^2 + 4(\alpha + \beta)/t_0^\gamma}$	$Y = \exp^{(x_0)} \exp \left[- \left(\alpha + \frac{\beta}{x_0} \right) t_1^{-\gamma} \right]$

Where: Y and Y_0 : variables at age t and t_0 , respectively; t and t_0 : age of stands (months); x : unobservable and unquantifiable theoretical variable; x_0 , l_0 and r_0 : initial solution for x ; A , B and Γ : parameters of the base model and; α , β , γ : global parameters of the dynamic equations.

$$\frac{N_a}{N_1} = \left(\frac{G_a}{G_1} \right)^\beta, \tag{3}$$

where f_{thin} is the response function to thinning, t and t_0 are the stand age (months), t_t is the age at which thinning occurred (months), N_t is the number of trees per hectare removed by thinning (trees per hectare); N_a is the number of trees per hectare remaining after thinning (trees per hectare), G_a is the remaining basal area of the stand ($m^2 \cdot ha^{-1}$); h_{dom_t} is the average dominant height before thinning (m); N_1 is the total number of trees before thinning (trees per hectare), G_1 is the stand basal area before thinning ($m^2 \cdot ha^{-1}$), and δ and β are the coefficients of the functions.

The response function to thinning depends on the dominant height; therefore, we needed a model that can estimate the dominant height at a particular time. We used a framework similar to the basal area growth, with the Lundqvist-Korf model as the base. For this approach the parameter B was expanded following the GADA methodology, by which the GADA is equivalent to the algebraic difference approach (ADA) (Prada et al., 2019):

$$h_{dom} = A \exp(-x^{-\Gamma}) \tag{4}$$

with

$$x = B$$

$$x_0 = -\ln\left(\frac{h_{dom_0}}{\alpha}\right) t_0^\gamma, \tag{5}$$

whose dynamic equation is

$$h_{dom} = \alpha \left(\frac{h_{dom_0}}{\alpha} \right)^{\left(\frac{\alpha}{\gamma}\right)}, \tag{6}$$

where: t and t_0 are the stand ages (months), h_{dom} and h_{dom_0} are the dominant heights (m) at ages t and t_0 (months), A , B and Γ are the coefficients of the base model; and α and γ are the global coefficients of the dynamic equation.

2.4. Total stand volume equation

The next step was to choose a model to estimate the wood volume at the total stand level. The Chapman-Richards model was used for the estimation:

$$V = A [1 - \exp(-Bt)]^\Gamma \tag{7}$$

The coefficients of asymptote (A) and growth rate (B) were expanded in the Chapman-Richards model (Eq. (7)), which were associated with stand variables (basal area and site index) that explain the variability of the volume production of a forest stand. Therefore, the basal area and site index are the covariates of the model (Eq. (8)).

$$V = (\alpha_0 G^{\alpha_1} S^{\alpha_2}) \{1 - \exp^{-(\beta_0 + \beta_1 S)^t}\}^\Gamma, \tag{8}$$

where t is the stand age (months), and V is the stand volume at age t ($m^3 \cdot ha^{-1}$), S is the stand site index (Base age at 120 months), G is the basal stand area ($m^2 \cdot ha^{-1}$); A , B and Γ are the model coefficients; and α_1 and β_1 are the coefficients related to the covariates G and S , used in the expansion of the coefficients A and B .

2.5. Model fitting, validation and comparison

For model fitting, we used the nonlinear generalized least squares method of the "gnls", function from the 'nlme' package of the R® software. We assuming an autoregressive structure of the residuals, whose models were fitted separately (García et al., 2011). To account for this possible autocorrelation, we modelled the error term using a first-order continuous autoregressive error structure (CAR (1)) that allows the models to be applied to irregularly spaced, unbalanced data (Grégoire et al., 1995; Zimmerman et al., 2001). The CAR (1) expands the error terms in the following way:

$$e_{ij} = \Psi_1 \rho^{t_{ij} - t_{ij-1}} e_{ij-1} + \varepsilon_{ij} \tag{9}$$

where e_{ij} is the j th ordinary residual on the i th individual (i.e. the difference between the observed and the estimated variable of plot i at age measurement j), $\Psi_1 = 1$ for $j > 1$ and it is zero for $j = 1$, ρ is the first-order autoregressive parameter to be estimated, $t_{ij} - t_{ij-1}$ is the time distance separating the j th from the $j - 1$ observations within each individual ($t_{ij} > t_{ij-1}$), and ε_{ij} is now the error term under conditions of independence and no heteroscedasticity.

For the fitted models, we constructed plots of the observed versus estimated values and percentage of residual dispersion, and a percentage residual histogram. The normality of the residuals was tested using the Shapiro-Wilk test (Shapiro and Wilk, 1965). The selection of the most accurate models was performed using the smallest square root of the mean percentage error ($RMSE_{\%}$) (Equation (10)), unbiased graphical analysis of the residuals, estimation efficiency, from the largest correlation coefficient between the estimated and observed values (R_{yy}) (Equation (11)), and the smallest Akaike's information criterion (AIC) (Equation (12)).

$$RMSE_{\%} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}} / \bar{Y} \times 100 \tag{10}$$

$$R_{yy} = \frac{\sum_{i=1}^n \hat{y}_i \cdot y_i}{\sqrt{(\hat{y}_i^2)(y_i^2)}} \tag{11}$$

$$AIC = 2K - 2\ln(L) \tag{12}$$

In the equations, y_i and \hat{y}_i are the observed and estimated values of the dependent variable, respectively; n is the total number of

observations; \bar{Y} is the mean of the dependent variable; K is the number of +1 parameters; e is the residual; and L is the likelihood ratio estimate.

For model validation, we used the Robinson et al. (2005) equivalence test (regression-based TOST using bootstrap) for 20% of the total number of plots measured in the continuous inventory. This test reverses the null hypothesis by determining the null hypothesis as an indicator of dissimilarity between the estimated and observed values. Equivalence tests have been used to compare means or similarities between estimates and observations on an individual basis (Robinson et al. 2005; Weiskittel et al. 2011). Since the regression approach to equivalence testing evaluates both, it is the most appropriate method (Robinson et al., 2005). Thus, we proceeded as follows: (1) the mean of the estimates was subtracted from all estimated values; (2) equivalence regions were established for the regression parameters (intercept and slope, with 25% at the 99% probability level, with 1000 bootstrapping); (3) linear regression was fitted between observations and estimated values; (4) equality for the intercept was tested by calculating the two bounds of the confidence intervals for the coefficient, which were then compared with the estimated equivalence region; (5) equality for the slope coefficient was tested by calculating the one-sided confidence interval for the slope coefficient, which was then compared with the estimated equivalence region; and (6) the dissimilarity hypothesis was accepted or rejected based on the test results.

Although Barrio Anta et al. (2006) and Castedo-Dorado et al. (2007) indicated that it was not necessary to directly quantify thinning in the stand-level basal area growth equations when the parameters were estimated using the GADA, we verified the need for the inclusion of the response function (f_{thin}) based on the equivalence test of Robinson et al. (2005). If the equivalence test indicates dissimilarity between the estimates of the basal area growth models in its reduced form (without the insertion of f_{thin}) and those in its complete form (with the insertion of f_{thin}), the thinning response function should be included, whereas if the

test indicates similarity, the reduced model is preferable.

2.6. Growth and production of teak trees

The data used to determine the teak annual increment were obtained from the results presented in the previous steps, whose initial basal area, which is used as the input for the projection of future basal area, was determined from the average basal area at the age of 24 months for each site class. The basal area and volume per hectare (Y) were obtained from the basal area growth and volume production models (Eq. (8)). The mean annual increment (MAI) was established as the projected volume divided by the age of the stand at age t .

$$MAI = Y/t \tag{13}$$

The current annual increment (CAI) was defined as the change in Y between the beginning and end of the growing season. Y_0 is the basal area or volume per hectare at age t_0 , and Y is the corresponding value at time t :

$$CAI = Y - Y_0 \tag{14}$$

Fig. 1 provides an overview of the study methodology.

3. Results

3.1. Basal area projection function

Table 3 contains the results of the statistical analyses for the evaluation and selection of the basal area projection models in their reduced and complete forms. The complete models showed a similar performance, whereby M1 and M2 provided $RMSE\%$ values slightly lower than model M3. The M1 model presented the lowest Akaike's information criterion, with M3 revealing the highest value. The Lundqvist-Korf base

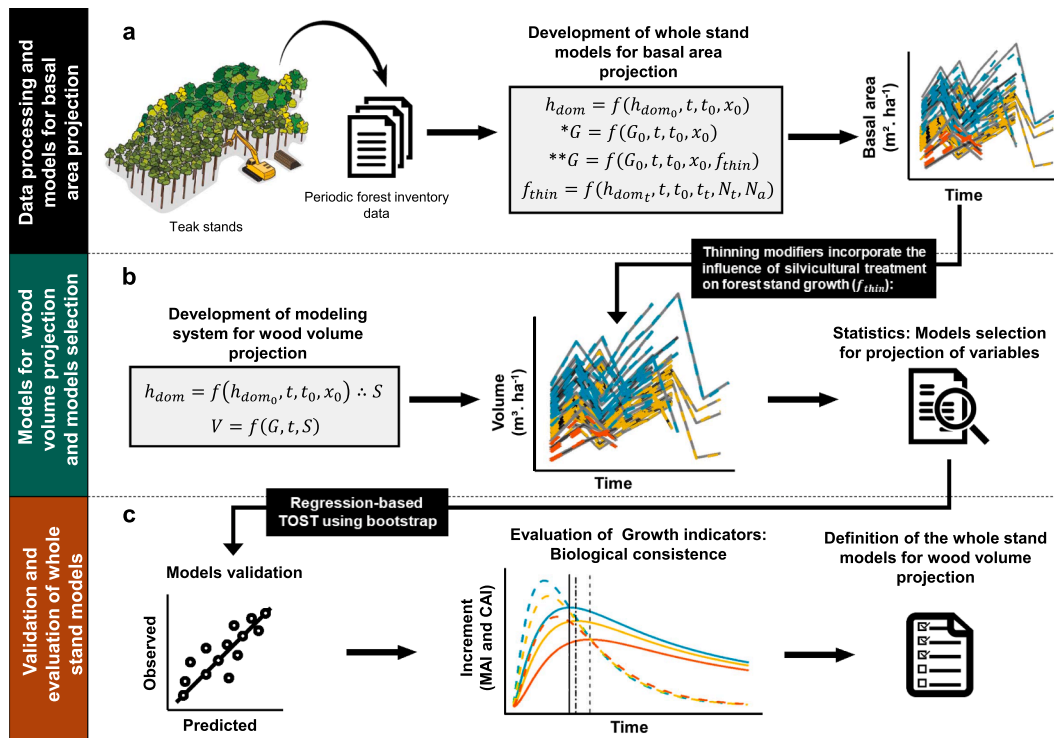


Fig. 1. Procedure for modeling growth and yield in a *T. grandis* clonal stand using the generalized algebraic difference approach in permanent plots, subjected to thinning. Data processing, modeling, validation and evaluation of models, represented by steps a, b and c. Where: G and G_0 is basal area at age t and t_0 ($m^2 \cdot ha^{-1}$); t and t_0 is stand age (months); h_{dom} and h_{dom_0} is dominant height at age t and t_0 (m); V is the predicted wood volume ($m^3 \cdot ha^{-1}$); x_0 is the theoretical non-quantifiable variable related to the site conditions (soil, climate and ecological conditions); S is the site index; f_{thin} is the response of growth in basal area to thinning; N_t is the number of thinned trees ($indiv \cdot ha^{-1}$); N_a is the number of remaining trees ($indiv \cdot ha^{-1}$); * GADA disregarding the effect of thinning on basal area growth and ** GADA considering the effects of thinning on basal area growth.

Table 3

Statistics of fit and precision for GADA applied to model the basal area ($\text{m}^2 \cdot \text{ha}^{-1}$) with and without the insertion of the thinning response function, in clonal teak stands, planted in Capitão Poço, Pará, Eastern Amazon.

Model	Parameters	$f_{\text{base}} \times f_{\text{thin}}$					f_{base}				
		Estimate	SE	RMSE%	R_{yy}	AIC	Estimate	SE	RMSE%	R_{yy}	AIC
M1	α	0.02352	<0.001	5.714	0.974	622.846	0.1018	<0.001	22.613	0.397	1347.867
	β	-17.3014	2.539				-153872.2	18864.71			
	γ	66.1610	8.476				410280.2	50393.65			
	δ	1.2315	<0.05				-	-			
	ρ	0.1089	-				4.63×10^{-9}	-			
M2	α	36.04347	1.238	5.938	0.973	638.562	16.66	0.0438	26.092	0.348	1423.729
	β	-0.00992	<0.001				-0.001	<0.001			
	γ	1.741	0.033				2.9	0.0569			
	δ	1.2297	<0.05				-	-			
	ρ	0.128	-				1.15×10^{-9}	-			
M3	α	-0.00656 ^{ns}	0.0103	6.651	0.972	694.623	60196.1	25868.9	26.429	0.335	1430.239
	β	65.79317	1.735				319343.7 ^{ns}	373205.8			
	γ	0.365844	0.029				2.6	0.2			
	δ	1.2435	<0.05				-	-			
	ρ	0.1409	-				0.00017	-			

Where: α , β , γ and δ : model parameters; RMSE%: square root of the mean percentage error; R_{yy} : correlation coefficient between estimated and observed values; AIC: Akaike's Information Criterion. ns: not significant at 95% probability level.

model showed a non-significant coefficient at the 95% probability level. The coefficient related to the thinning response function was highly significant in all models.

The fit statistics were inferior for the reduced models (Table 3),

whose RMSE% values were higher than those of all the complete models. The reduced models showed similar performance when compared to each other. The M3 model, fitted from the Lundqvist-Korf function, presented one non-significant coefficients at the 95% probability level.

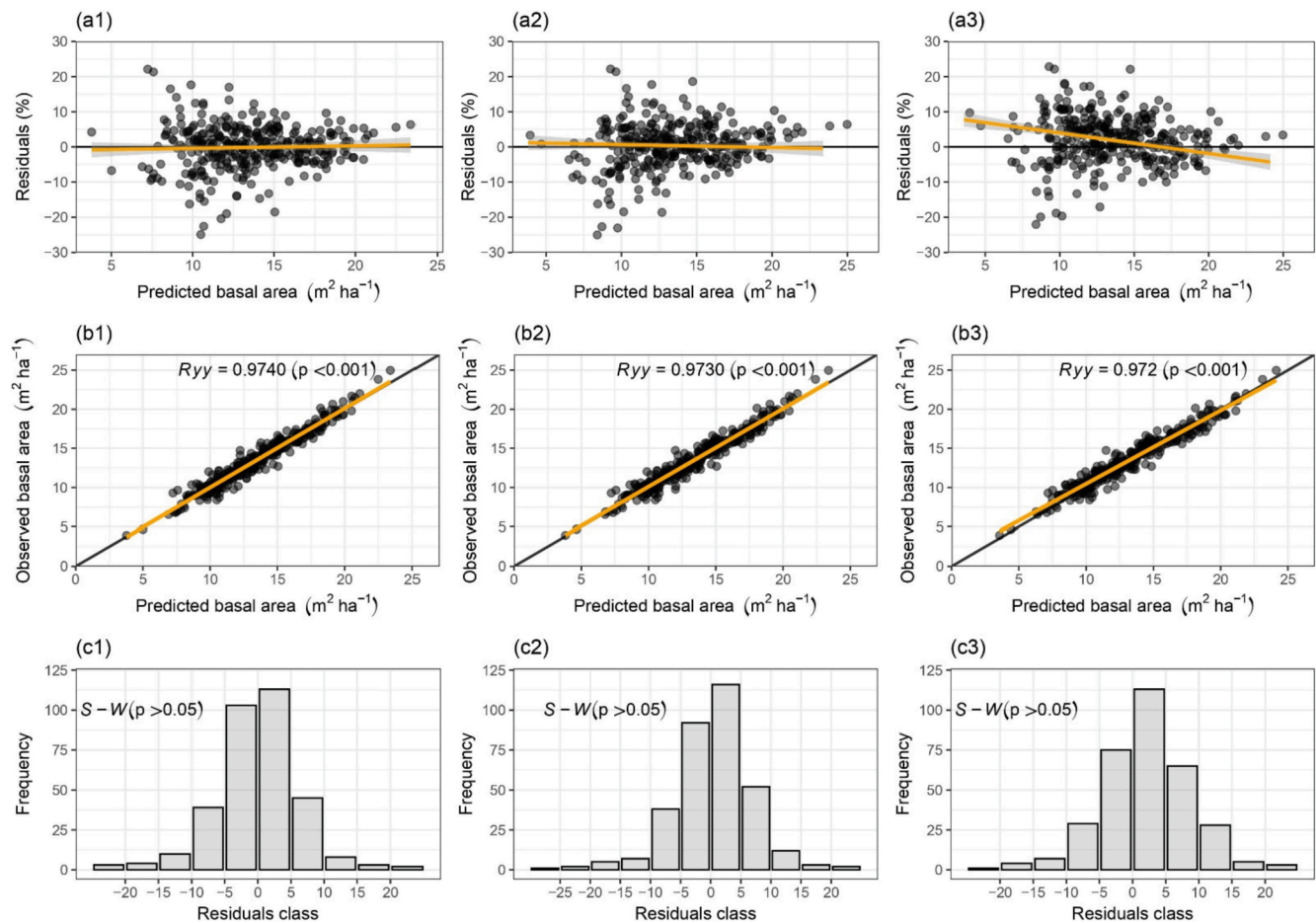


Fig. 2. Dispersion of residuals (ai), relation between observed x estimated (bi) and distribution of classes of residuals (ci) for equations of projection of the basal area considering the effect of thinning, in clonal teak plantations located in the limits of the Municipality of Capitão Poço, Pará, Eastern Amazon. Where: a1, b1 and c1: model M1; a2, b2 and c2: model M2 and a3, b3 and c3: model M3. S-W results of the Shapiro-Wilk test for residuals.

The estimation efficiency values were less than 0.4, indicating a weak correlation between the estimated and observed values.

The distribution of residuals showed normality and the absence of heteroscedasticity for all complete models. As observed in Fig. 2, the models fitted with the thinning response function presented normality, according to the Shapiro-Wilk test (p -value > 0.05), respectively. Models M1 and M2 showed similar trends, with a slight tendency to overestimate the basal area at lower values and underestimate it at higher values. Despite this, the residuals were mainly scattered near zero. The strong correlation between the estimated and observed values indicates the high predictive ability of the fitted models. The histograms of the residuals showed a higher frequency in classes close to zero.

The reduced models tended to underestimate the basal area in unthinned plots and overestimate basal area in the thinned regions (Fig. 3); therefore, the residuals did not have a normal distribution according to the Shapiro-Wilk test (p < 0.05). To determine the trends between the approaches, residual scatter plots were presented at different scales.

3.2. Dominant height, volume projection, and thinning controller function

The fitted models were accurate for the dominant height growth and volume production projection (Table 4). The Lundqvist-Korf model had an $RMSE\%$ of 4.74% and a correlation coefficient greater than 0.9, indicating a strong correlation between the estimated and observed dominant heights. The Chapman-Richards model for the projection of volumetric yield presented an $RMSE\%$ of less than 3%, indicating high accuracy, and a correlation coefficient greater than 0.9. The value for

the coefficient related to the thinning controller was close to 1, indicating that the proportion between the number of trees and the basal area thinned was close; therefore, the removal of 50% of the trees would result in an approximate 50% reduction in basal area.

Regarding the analysis of residuals, the dominant height model showed tendencies to overestimate the values in the smallest dimensions, while the volumetric production projection model tended to underestimate the wood volume at values below $50 \text{ m}^3 \cdot \text{ha}^{-1}$. As shown in Fig. 4, both models presented normality for the residuals, according to the Shapiro-Wilk test (p -value > 0.05). The strong correlation between the estimated and observed values indicates the high predictive ability of the fitted models. The histograms of the residuals showed a higher frequency in classes close to zero. Thus, the Lundqvist-Korf model was used for site classification (base age at 120 months), which presented biological logic and polymorphic site index curves with a two meter amplitude between yield classes (SIII, 15.5; SII, 17.5; SI, 19.5).

3.3. Validation of the models

The equivalence tests, represented in Table 5, showed that the equations fitted for the basal area from the complete models were validated, with the exception of model M3, which presented a non-significant coefficient; therefore, it was not subjected to the validation test. The equivalence test ($\epsilon = 0.25$ and $\alpha = 0.01$, with 1000 bootstrappings) indicated that the estimated and observed values were not statistically different, as the confidence interval was within the region of similarity, so the dissimilarity hypothesis was rejected for models M1 and M2. The same results were observed for the dominant height growth

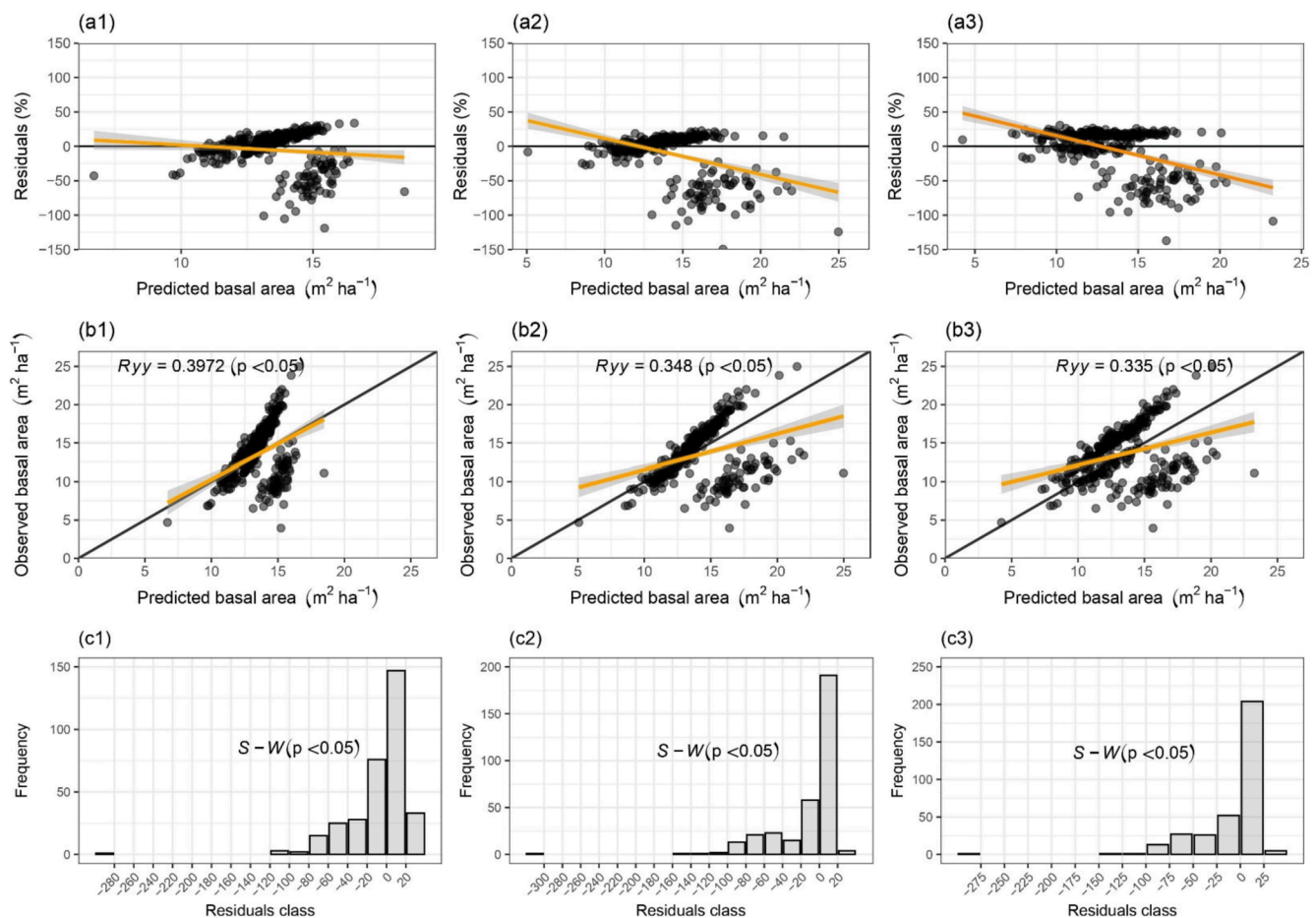


Fig. 3. Dispersion of residuals (ai), relation between observed x estimated (bi) and distribution of classes of residuals (ci) for equations of projection of the basal area considering the effect of thinning, in clonal teak plantations located in the limits of the Municipality of Capitão Poço, Pará, Eastern Amazon. Where: a1, b1 and c1: model M1; a2, b2 and c2: model M2 and a3, b3 and c3: model M3. S-W, results of the Shapiro-Wilk test for residuals.

Table 4

Statistics of fit and precision for modeling growth in dominant height and projection of volumetric production, in clonal teak stands, planted in Capitão Poço, Pará, Eastern Amazon.

Model	Parameters	Estimate	SE	RMSE%	R_{yy}	AIC
Lundqvist-Korf	α	25.1941	1.5567	4.743	0.957	725.882
	γ	0.60004	0.0451			
	ρ	1.42×10^{-9}	–			
Chapman Richards – Volume	α_0	2.0624	0.2796	2.212	0.996	1171.97
	α_1	1.0046	0.0043			
	α_2	0.4590	0.0470			
	β_0	–0.0349	0.0058			
	β_1	0.0038	<0.001			
	Γ	0.6005	0.0388			
	ρ	0.1443	–			
Thinning controller	β	0.9981	0.0021	0.6459	0.999	–2168.33

^aWhere: $\alpha, \beta, \gamma, \Gamma$: model parameters; $RMSE\%$: square root of the mean percentage error; R_{yy} : correlation coefficient between estimated and observed values; AIC: Akaike Information Criterion.

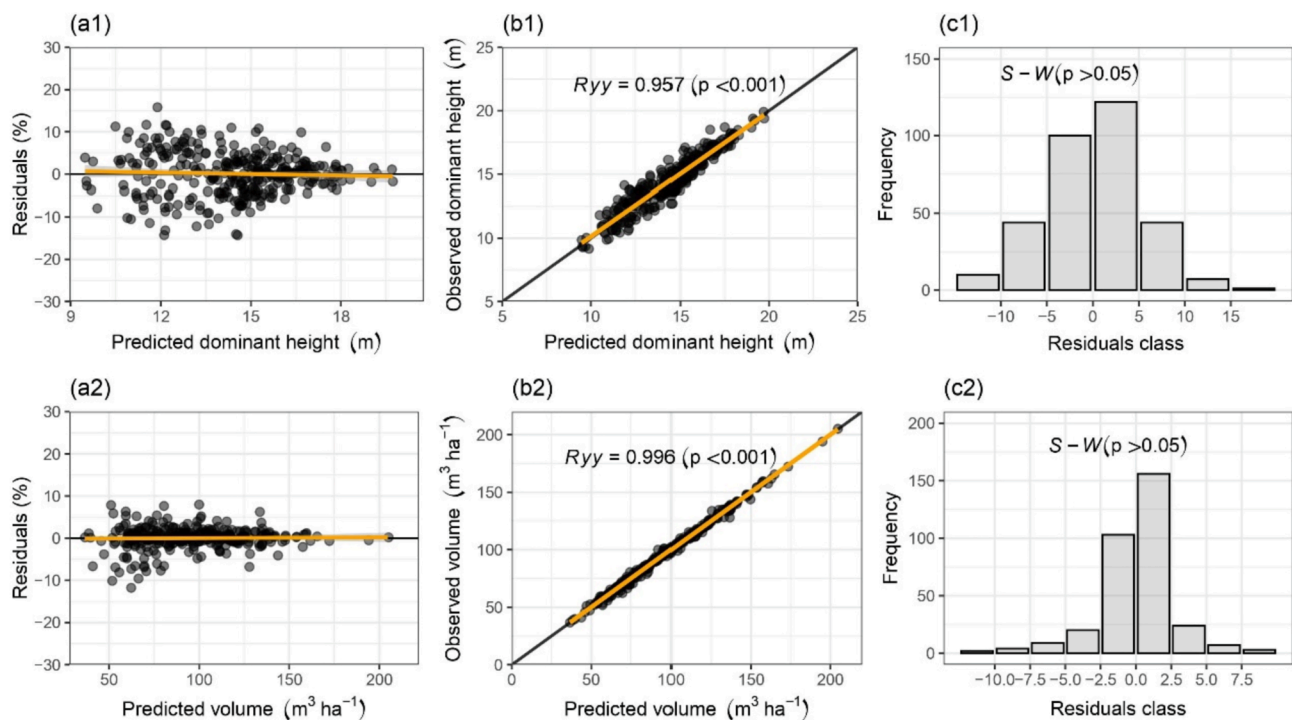


Fig. 4. Dispersion of residues (ai), relation between observed x estimated (bi) and distribution of residue classes (ci) for equations of projection of the basal area considering the effect of thinning, in clonal teak plantations located in the limits of the Municipality of Capitão Poço, Pará, Eastern Amazon. Where: a1, b1 and c1: dominant height growth model; a2, b2 and c2: model for projection of volumetric production. S-W and D-W, results of the Shapiro-Wilk test.

Table 5

Validation test for the selected models, for clonal teak stands subjected to thinning, planted in Capitão Poço, Pará, Eastern Amazon.

Approach	Models	Parameters	Confidence Interval	Similarity region	Dissimilarity
Validation	M1	Intercept	12.928 ± 13.202	12.748 ± 13.248	Rejected
		Slope	0.977 ± 1.048	0.75 ± 1.25	Rejected
	M2	Intercept	12.913 ± 13.201	9.737 ± 16.229	Rejected
		Slope	0.971 ± 1.040	0.75 ± 1.25	Rejected
	Dominant height	Intercept	13.978 ± 14.465	10.655 ± 17.756	Rejected
		Slope	0.909 ± 1.097	0.75 ± 1.25	Rejected
	Volume	Intercept	91.295 ± 91.603	91.103 ± 92.203	Rejected
		Slope	0.981 ± 1.011	0.75 ± 1.25	Rejected
Comparison	M1 _{thinning}	Intercept	12.606 ± 13.469	9.745 ± 16.243	Rejected
	× M1 _{unthinning}	Slope	0.475 ± 0.777	0.75 ± 1.25	Not rejected
	M2 _{thinning}	Intercept	12.886 ± 13.928	9.737 ± 16.229	Rejected
	× M2 _{unthinning}	Slope	0.547 ± 0.831	0.75 ± 1.25	Not rejected

Where: M1_{thinning} and M2_{thinning}: Basal area models considering thinning; M1_{unthinning} and M2_{unthinning}: Basal area models without considering the effect of thinning.

and volume production projection models.

To compare the differences between the basal area growth models in their complete and reduced forms, the equivalence test was used, which revealed differences in the estimates ($\epsilon = 0.25$ and $\alpha = 0.01$, with 1000 bootstrappings). Thus, given the results of the fit statistics (Table 3) in conjunction with the equivalence test, the thinning response function should be included in basal area growth models for teak plantations that undergo heavy thinning.

3.4. Growth and yield

Fig. 5 represents the production curves and the estimates of the mean (MAI) and current (CAI) annual increments of clonal teakwood stands for basal area and volume for site classes 15.5, 17.5, and 19.5. The initial basal area, which was the input for projection of future basal area, was determined from the mean basal area at the age of 24 months for each site class, with $4.65 \text{ m}^2 \cdot \text{ha}^{-1}$ for Site 15.5; $6.54 \text{ m}^2 \cdot \text{ha}^{-1}$ for Site 17.5 and $7.44 \text{ m}^2 \cdot \text{ha}^{-1}$ for Site 19.5. The projection of the basal area was performed for the model M1 with the higher accuracy, and for each site class. The modeling for growth and yield at stand-level were composed of the following combinations of equations: model M1 for projection of basal area (integrated with the thinning response function); model for projection of dominant height (Eq. (6)) and; model for projection of stands volume (Eq. (7)).

The models generated intervention ages consistent with the biological growth theory. Fig. 5b2 reveals that for the wood volume variable, intervention periods occurred at 56 months for Site 19.5, 63 months for Site 17.5, and 76 months for Site 15.5. For the basal area, the intervention periods were earlier, between the 44th and 60th months. The projections presented in Fig. 5a1 and 5a2 were performed without thinning simulations; the thinning response function assumed a value equal to 1. On the other hand, in Fig. 5c1 and 5c2, were performed thinning simulations; in which, removed 50% of basal area on period that MAI reaches its maximum for basal area. Two thinning simulations were performed.

4. Discussion

4.1. Basal area

Overall, the Chapman-Richards base model showed slightly better fit results than the others, which is similar to the results from a number of other studies, such as Barrio-Anta et al. (2008), González-García et al. (2015), and Prada et al. (2019). However, Hossfeld's model can be used in the prognosis of teak basal area growth based on the results observed in the validation.

The transition functions for basal area prognosis were derived using the generalized algebraic difference approach (GADA), which ensures that compatibility properties provide consistent estimates (Prada et al., 2019). By quantifying the effect of thinning on basal area growth from a modifier function, more accurate adjustments were obtained that avoided greater tendencies to under- or overestimate the basal area in unthinned and thinned plots, respectively. This greater precision was confirmed by the equivalence test (Table 5), which indicated differences in the estimates between the models with and without thinning. Our results contrasted from those observed by Barrio Anta et al. (2006) and Castedo-Dorado et al. (2007), who modeled the growth in basal area for different species of the genus *Pinus* in natural stands with thinning intensity ranging from 10% to 50%, and found no requirement to include the effect of thinning on growth, as no differences were observed in relation to the this process. Therefore, it is evident that the inclusion or exclusion of the influence of thinning in models of growth and production depends on many factors, including the sensitivity of the species to silvicultural practices.

The various thinning modifiers developed in other studies (Gyawali and Burkhart, 2015; Kuehne et al., 2016), have also shown significant improvements in growth projections for the basal area at the stand level. Therefore, thinning response functions appear to be important for projecting forest growth and ensuring accurate adjustments. However, the gains in accuracy were more significant in our study when compared to the works of Kuehne et al. (2016) and Gyawali and Burkhart (2015),

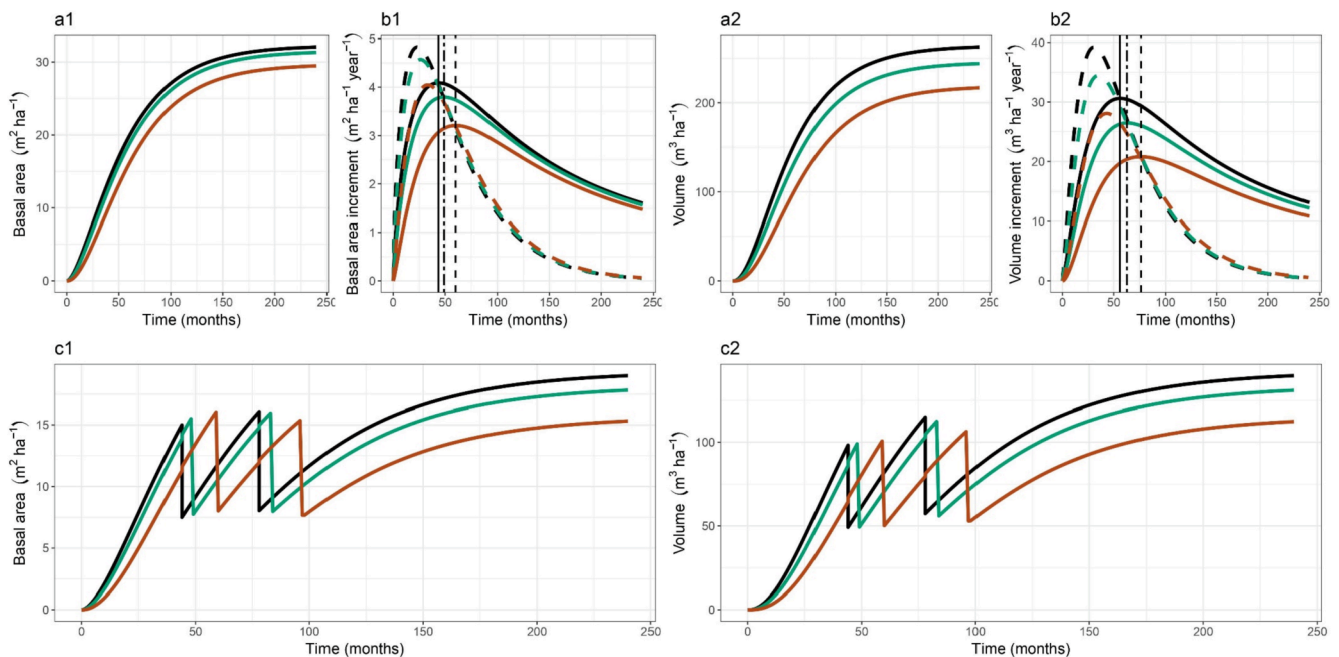


Fig. 5. Production, average annual increment (MAI), and current annual increment (CAI) per hectare, for different site classes. Dashed line represents CAI per hectare, solid line represents MAI per hectare (b1 and b2). Black lines indicate site class 19.5; green lines represent site class 17.5; and orange lines represent site class 15.5. Vertical black lines represent the time at which MAI reaches its maximum, solid line = intervention age for site 19.5 m; dotted line = intervention age for site 17.5 m and; dashed line = intervention age for site 15.5 m. a1 and a2 represent the projections for basal area and volume, respectively, and; b1 and b2 represent the mean and current annual increments. In c1 and c2, are shown thinning simulations; in which, removed 50% of basal area on period that MAI reaches its maximum for basal area. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

whose $RMSE_{\%}$ values were reduced by approximately 20% when considering the influence of the thinning modifier.

Theoretically, when a forest stand is thinned, its growth characteristics change, and several studies have shown that basal area growth rates of thinned stands exceed those of unthinned plots (e.g., Seta et al., 2021; Zapata-Cuartas et al., 2021). As teak is a shade-intolerant species, light intensity is one of the most important production factors for expressing full growth potential, especially radial growth (Budiadi et al., 2017). Therefore, thinning is a way to modify the resource allocation process of the forest site by concentrating production factors (light, water, and nutrients) on trees that have higher added value (Ding and Zang, 2021; Weiskittel et al., 2011; Xie et al., 2020).

For clonal teak plantations, thinning should be intense and frequent to maintain the diametric growth of the selected clones, which can be rapid in the early growth stage ($>2.5 \text{ cm year}^{-1}$) (Seta et al., 2021). For example, in clonal teak plantations in Indonesia at 3 years after the first thinning, a reduction in stand density of 50% promoted significant differences in diameter growth compared to moderate thinning (25%) and the control (0%) (Budiadi et al., 2017). From this perspective, since thinning affects species growth, it is important that its influence be incorporated into growth and production models, and aid forest managers to present more accurate decisions regarding the intensity and frequency of management interventions.

In this study, a multiplicative thinning modifier was proposed at the total stand level to predict the direct effect of thinning on basal area growth. This approach is different from that adopted by Torres et al. (2020), who simulated the growth and production of wood volume and biomass of thinned teak plantations using SSA by adding the thinned value from the previous period to the following period to avoid underestimating the total production. The modifier proposed in this study depends on the age at which thinning occurred, the intensity of thinning, and the dominant height prior to silvicultural practice, which distinguishes it from the modifier term presented by Pienaar and Shiver (1986). Thus, the three-dimensional equation ($Y = f(t, x)$) that describes both cross section and longitudinal changes with two independent variables, t and x , is expanded with the insertion of the thinning modifier term as a multiplier ($Y = f(t, x, f_{thin})$), thus allowing the growth function to capture the changes that occur in the stand due to the intervention of thinning.

Trees compete for a variety of resources, including light, water, nutrients, and physical space. As trees grow, they modify their surrounding environment, which alters the ability of their neighbors to acquire resources (Weiskittel et al., 2011), and after thinning, the remaining trees show an increase in growth rate due to reduced competition and an increase in resource availability (Seta et al., 2021). The differences observed in tree growth are largely results of the variation and heterogeneity of factors related to the forest site, such as water, light, temperature, and nutrients that influence the formation and allocation of biomass, which are processes mediated by meristems (Mathieu et al., 2008; Tondjo et al., 2018).

Thus, meristematic activity depends on site conditions, and more specifically on the duration of dry seasons for tropical species such as teak (Tondjo et al., 2018). Including the dominant height prior to thinning in f_{thin} allows for the capture of the remaining forest growth for different productive capacity classes, as dominant height is an effective integrator of key biological growth determinants (Kuehne et al., 2016; Scolforo et al., 2020; Weiskittel et al., 2011) that is less affected by thinning, and is an indicator of site productivity (Burkhart and Tomé 2012; Kuehne et al. 2016; Scolforo et al. 2020). Although essential to explain growth and production, the effect of thinning on plant physiology was not evaluated in this study. Despite this, tree sap flux density significantly increased after thinning, but the effect is usually temporary for this species (Sinacore et al., 2019).

4.2. Dominant height, volume, and thinning controller

The Lundqvist-Korf model provided important features for modeling the dominant height, such as polymorphism (single asymptote) and biological logic for site classification. The selected equation provided a site index with class centers ranging from 15.5 to 19.5 m, which are within the range of values estimated by Kenzo et al. (2020) of 10 to 32 m for plantations of different ages in regions of Thailand. In naturally occurring, seedling-originating plantations, dominant trees can reach an average of 70 cm in diameter and 40 m in height at 50–80 years of age, while 40 years of age the dominant trees are up to 35 m in height and 60 cm in diameter (Kollert and Kleine, 2017).

Some authors have observed higher values for other regions of Latin America including Cañadas-L et al. (2018), who observed, values ranging from 15 to 23 m at 120 months of age in the productive sites of Los Ríos province, Ecuador, while Bermejo et al. (2004) reported values from 19 to 23 m (reference age of 120 months) for Guanacaste Province, Costa Rica. In this study, the site index ranged from 15.5 to 19.5 m with a reference age of 120 months, and areas with an index below 17 m in the planting area can be considered to have poor ecological conditions for economic teak cultivation, as found by Cañadas-L et al. (2018).

Although the Lundqvist-Korf model was accurate ($RQME_{\%} < 6\%$) and presented biological logic in the estimates of dominant height, it has been reported by other studies (Cañadas-L et al., 2018; Cieszewski and Strub, 2008; Tewari et al., 2014) that site index models with polymorphic multiple asymptote curves are more successful in fulfilling all desirable properties of growth patterns than anamorphic or polymorphic single asymptote models. However, common asymptotes for site curves may be irrelevant, since the behavior of growth curves can be adequate within the age ranges used in practice, and naturally attained patterns at older ages (Diéguez-Aranda et al., 2005).

For stand-level volume modeling, we used the Chapman-Richards function, which is a generalization of the von Bertalanffy function that describes the absolute growth rate as the difference between an anabolic rate (constructive metabolism), which in plants is proportional to the photosynthetically active area, and the catabolic rate (destructive metabolism), which is proportional to biomass (Burkhart and Tomé, 2012). Depending on environmental and competitive factors, the model parameters can change over time and yield different growth rates (Pommerening and Muszta, 2016). The Chapman-Richards model was modified with the covariates site and basal area. Thus, by inserting such variables into the model, it was possible to capture the environmental and competitive effects in the projection of volumetric production. Since the basal area is present in the model, the effect of thinning is implicit in the volume estimates.

Restrepo and Orrego (2015) used the von Bertalanffy model with four fixed effect parameters, two random effect variables, and one environmental covariate in the model of a teak stand in Colombia, and obtained accurate fits. Thus, the addition of covariates allows for gains in estimation accuracy, and also enables the generation of polymorphic production curves, which express differentiation rather than proportionality in volume growth, whose growth rate in the model fitted in this study is dependent on forest site factors.

The Chapman-Richards model has expansion and decline components that are associated with the parameters A , B and L for expansion, and the coefficients A and B for decline (Zeide, 1993). Therefore, the parameter A represents the maximum value of stored volume to be reached by the stand when age tends to infinity (Burkhart and Tomé, 2012; Mayoral et al., 2019; Prada et al., 2019; Salekin et al., 2020; Xu et al., 2020); hence, by inserting the future basal area and site index as covariates, multiple asymptote values are obtained, which are implicitly modified by thinning and affect basal area growth. In contrast, the coefficient B represents the growth rate of wood volume (Mayoral et al., 2019; Mendonça et al., 2017; Passos dos Santos et al., 2020); therefore, with the insertion of the site index covariate, it was possible to generate a model with multiple growth rates, which responded to the influence of

the productive capacity of the site, whereby higher values for B represented higher growth rates.

The relationship between the proportions of thinned trees and thinned basal area was equivalent, and the parameter of the thinning control function was close to 1. We assume that this result is a consequence of the homogeneity of the plantation, which is of clonal origin. Despite this, thinning is recommended, because when considering the timing, controlling the use intensity by the number of trees may not be recommended in some cases, since the result of the intervention depends on the type of thinning (selective or systematic). If thinning is performed from below, the crown density of the remaining trees will be more pronounced than if the same number of large trees is removed, so thinning intensity is better controlled by stipulating the amount of basal area to be removed (% or m^2) (Campos and Leite, 2017).

4.3. Growth and yield

Growth is characterized by changes in trunk shape and size, with the continuous addition of new layers of woody material (Campos and Leite, 2017). Thus, several factors strongly influence the growth process, such as site quality and the site occupation level, whereby higher productive capacity results in greater growth and competition for the same initial number of trees and spatial arrangement, thereby promoting a lower cutting age (Burkhardt and Tomé, 2012; Pommerening and Muszta, 2016). Such a pattern was observed for the simulation of the selected models, corroborating the ability of this approach to provide intervention ages within biological growth assumptions.

In this study, the projections made from the selected models resulted in intervention ages ranging between 56 months and 76 months, which are within the ranges proposed in the literature of 36 to 96 months of age (Cañadas-L et al., 2018; Kollert and Kleine, 2017). For plantations with a higher density of individuals, Vigulu et al. (2019) observed a reduction in growth rate between 40 and 60 months, indicating the beginning of competition for resources and the need for thinning.

The average annual increments estimated by the models were close to $31 \text{ m}^3 \cdot \text{ha}^{-1} \cdot \text{year}^{-1}$, which is considered excellent for teak. These values are similar to those observed by Kollert and Kleine (2017) in regions of Brazil and Mexico, with increments of up to $32 \text{ m}^3 \cdot \text{ha}^{-1} \cdot \text{year}^{-1}$ for clonal plantations. Similar to the age of intervention, the average increments estimated by the models were also close to those observed for planting, with an average of $29 \text{ m}^3 \cdot \text{ha}^{-1} \cdot \text{year}^{-1}$ at the best site.

Thus, the models generated from the GADA dynamic derivation techniques with the thinning response function provided realistic intervention ages with the growth conditions of the species and the plantation in question (56 to 76 months), showing that models with biological properties provide predictions of growth and forest production with a high degree of reliability (Cañadas-L et al., 2018; Tewari and Singh, 2018). Therefore, a first thinning should be conducted before the aforementioned periods, to avoid competition and its consequences.

The models presented in this study are important for forest managers because they allow the simulation of different productive scenarios through the influence of thinning, and can guide forest managers toward the ideal point of intervention, minimizing subjective decisions in teak management. In addition, they aim to minimize the possible harmful impacts of poorly planned thinning, because when applied at the correct time and intensity, it can alter the post-thinning growth in relation to pre-thinning tendencies. Poorly executed interventions can compromise future production, reduce productivity, and result in economic losses (Seta et al., 2021).

5. Conclusions

There is a need to consider the effect of thinning, through a response modifier, in dynamic models for teak stands in the eastern Amazon region. The models for basal area growth that are adjusted using the GADA

methodology, provided accurate, consistent, and valid adjustments when thinning was integrated into the dynamic equation of a forest plantation sensitive to silvicultural treatments. The selected model meets the premises for the development of equations of growth and forest production, demonstrating the polymorphism, multiple asymptotes, and biological logic of the dynamics of teak plantations in the region.

The developed thinning response modifier described important characteristics for the proposed growth and production model, considering the age at which the stand is thinned, and the ability of the stand to resume growth, represented by the dominant height. The estimated basal area served as a covariate for the prognostic model and the prediction of volume per unit area. Besides providing multiple asymptotes and polymorphism, this association of variables revealed the most accurate description of the age of intervention, with performance compatible with the biological growth observed in the analyzed stands. The models developed is consistent for teak stands in the Amazon and can be useful for defining the technical age of rotation and productivity of teak plantations in the region.

CRedit authorship contribution statement

Hallefy Júnio de Souza: Conceptualization, Methodology, Investigation, Software, Formal analysis, Writing – original draft, Visualization, Validation. **Éder Pereira Miguel:** Conceptualization, Methodology, Formal analysis, Writing – review & editing, Supervision, Validation. **Rodrigo Geroni Mendes Nascimento:** Software, Formal analysis, Writing – review & editing, Validation. **Christian Dias Cabacinha:** Writing – review & editing, Formal analysis. **Alba Valéria Rezende:** Writing – review & editing, Formal analysis. **Mário Lima dos Santos:** Conceptualization, Data curation, Investigation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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